A REGULAR HOT-POWDER COMPRESSION REGIME

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We propose a new analytical solution for the problem familiar from the theory of hot compression in connection with the axial unilateral compression of a viscous porous material in a cylindrical mold.

Interest in the problem of axial unilateral compression of a viscous porous material has been generated both by the development of the technology of hot pressforming methods and the study of high-temperature rheology in powder materials. In its initial stage, this was treated under an assumption of density uniformity and an absence of friction at the walls [1, 2]. The analytical solution derived thereby was utilized as a basis for the selection of viscosimetric and rheological variables and the establishment of a method a priori to determine the unknown properties of the material and the parameters governing their dependence on density [3]. The studies [4, 5] are devoted to investigations into the unique features encountered in this type of deformation, in conjunction with the nonuniformity of density distribution, and the method of characteristics was used in these studies to derive a system of integrodifferential equations to determine the density of a material and its flow velocity, and here the authors also established certain important quantitative relationships governing the compression of a material for a particular type of deformation. In the following we analyze the analytical solution of the problem under consideration in Lagrange variables. This allows us more clearly to interpret the derived results and to establish new quantitative relationships with respect to compression, thus expanding the physical concepts relative to this process. In particular, among such results we must include the fact that within the framework of the adopted assumptions we are confronted with a regular compression regime. It is demonstrated that the fundamental operational characteristic of the process, i.e., the relationship between the speed of the piston and the applied force for fixed instants of time is nonmonotonic in nature, which is a result of the competition between the effect of the load factors and the volumetric viscosity that is dependent on density.

Formulation of the Problem. Let us examine the unilateral compression of a viscous porous material in a cylinder closed at the bottom ($z = 0$) and bounded on the top [$z = H(t)$] by means of a moving piston. The normal forces N(t) applied to the piston generates stresses σ_r, σ_Θ, and σ_z within the material. The force of friction against the walls of the mold can be neglected. We will adopt the hypothesis of plane cross sections, according to which we have only a single nonzero velocity component $v_z = v \neq 0$ in the material.

For the description of the selected type of flow in a compressed material we find in [1] a system of continuity equations and equations of motion, in conjunction with rheological relationships, such as those that are extensively used in the theory of hot compression. In the following we will examine the system in Lagrange coordinates, related to particles fixed within the medium under consideration, although in motion. The utilization of these coordinates in the description of the motion of compressed materials frequently allows us to derive analytical solutions for the problems and establishes certain advantages in the interpretation of the results. As in the case of gasdynamics [6], as our Lagrange coordinates it is convenient to choose the time τ and the quantity q, equal to the mass of the material located between the cross sections $z = 0$ and $z = z$:

$$\tau(z, t) = t, \quad q(z, t) = \int_0^z \rho(z, t)\, dz.$$

Let us note that $M = \int_0^{H(t)} \rho(z, t)\, dz$ determines the entire mass of the billet exhibiting a height H(t).

When we take into consideration the empirical relationship between the shearing viscosity η and the volumetric viscosity ξ from [7] as functions of the density, namely:

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$$\eta = \eta_1 \rho^m, \quad \xi = \frac{4}{3} \eta(\rho) \rho/(1-\rho) \tag{1}$$

the original equations are written in Lagrange form as follows:

$$\frac{\partial \rho}{\partial \tau} + \rho^2 \frac{\partial v}{\partial q} = 0, \tag{2}$$

$$\frac{\partial \sigma_z}{\partial z} = 0, \tag{3}$$

$$\sigma_z = \frac{4}{3} \eta_1 \frac{\rho^{m+1}}{1-\rho} \frac{\partial v}{\partial q}, \tag{4}$$

$$\sigma_\theta = \sigma_r = \frac{2}{3} \eta_1 \rho^{m+1} \frac{3\rho - 1}{1-\rho} \frac{\partial v}{\partial q}. \tag{5}$$

In order to solve the original system of equations we have to specify the initial distribution of the density with respect to the mass coordinate q:

$$\tau = 0, \quad \rho = \rho_0(q) \tag{6}$$

and the boundary conditions. At the lower boundary of the compressed material we assume a condition of adhesion

$$q = 0, \quad v(0) = 0. \tag{7}$$

At the upper boundary, depending on the deformation regime, we must distinguish two types of boundary conditions:

$$q = M, \quad \sigma_z = -N(\tau) \tag{8a}$$

for a regime with specified force and

$$q = M, \quad V = V(\tau) \tag{8b}$$

for a regime with a specified speed of piston displacement. Let us examine these regimes separately from one another.

Specified Force Regime. In this case the axial stress is defined entirely by the pressure $N(\tau)$ applied from without and, according to (3), it does not change over the height of the billet: $\sigma_z = -N(\tau)$. Expressing $\partial v/\partial q$ in terms of (4) and then substituting it into the continuity equation (2), we obtain a kinetic equation for the compression process

$$\frac{\partial \rho}{\partial \tau} = -\frac{4}{3\eta_1} N(\tau) \frac{1-\rho}{\rho^{m-1}}. \tag{9}$$

This might serve both for purposes of calculating the density distribution $\rho(q, \tau)$ and to solve various kinds of reciprocal problems such as, for example, estimating the compression time or the viscosity η_1 of the solid phase for a known distribution of density. Let us take note of the fact that according to (9) the rate of compression with a nonuniform initial distribution of density is clearly independent of the mass coordinate q and is represented in the form of a product of functions where one of the functions is dependent on the time τ while the other is dependent on the density ρ. Precisely this form of the kinetic compression equation was derived in [2] for the case of uniform density. From this emanates important consequences with respect to the nature of the compression process. The compression of any isolated individual volume corresponding to the coordinate q and exhibiting an initial density $\rho_0(q)$, given a nonuniform initial distribution of density with respect to the coordinate q, will proceed in a manner identical to that of a uniform density distribution exhibiting the same initial density ρ_0. Such a compression regime may be referred to as regular, in analogy with a regular thermal regime [8] in which the effect of the initial distribution of temperature on the relationship to time on the part of the temperature within the body makes itself apparent only as a dimension and has no effect on the actual nature of this relationship. If we examine the relative rate of compression

$$v_\rho = \frac{\rho^{m-1}}{1-\rho} \frac{\partial \rho}{\partial \tau}, \tag{10}$$

then in Lagrange coordinates this characteristic will depend only on time and will be independent of the initial distribution of density, regardless of the form of its nonuniformity. When we introduce the notation $J = \int\limits_{\rho_0}^{\rho} \frac{\rho^{m-1}}{1-\rho}\, d\rho$, it becomes possible to rewrite Eq. (9) in the form

$$J[\rho(q, \tau)] = -\frac{3}{4\eta_1}\int\limits_0^{\tau} N(\tau)\, d\tau. \tag{11}$$

In relationship (11), on the right-hand side, we have the familiar time function. However, there is no difficulty in finding the integral $J(\rho)$ on the left-hand side. For whole m this integral is found analytically. In particular, for $N(\tau) = N_0$, m = 2, and m = 3 it is easy, analytically, to obtain the following relationship from (11) for the density of the material as a function of time:

$$\bar{\tau} = \ln \frac{1-\rho_0}{1-\bar{\rho}} - \bar{\rho} + \rho_0, \tag{12}$$

$$\bar{\tau} = \ln \frac{1-\rho_0}{1-\bar{\rho}} - (\bar{\rho} - \rho_0) - \left(\frac{\bar{\rho}^2 - \rho_0^2}{2}\right). \tag{13}$$

These expressions coincide in accuracy with the interpolation formulas proposed in [3] for corresponding m where the average integral density $\bar{\rho} = \int_0^t \rho(\bar{z})d\bar{z}/\bar{z}$ ($\bar{z} = H/H_0$) is a function of the dimensionless time $\bar{\tau} = 3N_0\tau/4\eta_1$, obtained through processing of numerical results.

Let us define the velocity field. Since the axial stress does not change through the height of the compression, from (4) we find the velocity gradient:

$$\frac{\partial v}{\partial q} = -\frac{3}{4\eta_1}\frac{1-\rho}{\rho^{m+1}} N(\tau). \tag{14}$$

With consideration of the boundary condition at the lower end of the compression, in the integration of (14) we determine the velocity of material flow

$$v(q, \tau) = -\frac{3}{4\eta_1} N(\tau) \int\limits_0^q \frac{1-\rho}{\rho^{m+1}}\, dq. \tag{15}$$

Assuming that q = M, from (15) we find the relationship which links the operational characteristics of the compression process, i.e., the speed of the piston and the force exerted on the piston:

$$V(\tau) = -\frac{3}{4\eta_1} N(\tau) F(M), \tag{16}$$

where $F(M) = \int\limits_0^M [(1-\rho)/\rho^{m+1}]\, dq$. The determination of this integral is associated with the preliminary determination from (11) of the function $\rho(q, \tau)$. Let us find the distribution of the density and the velocity of the material under certain simplifying assumptions which are of practical significance. For a qualitative analysis we can limit ourselves to simple forms of the relationship between the shearing viscosity and density. For example, if the shearing viscosity is linearly dependent on density (m = 1), for porosity $\Pi = 1 - \rho$ and velocity $V(\tau)$ from (11) and (16) we, respectively, obtain

$$\Pi(q, \tau) = \Pi_0(q) \exp[-A(\tau)], \tag{17}$$

$$V(\tau) = -\frac{3}{4\eta_1} N(\tau) \int\limits_0^M \frac{\Pi}{(1-\Pi)^2}\, dq, \tag{18}$$

where $A(\tau) = \frac{3}{4\eta_1}\int\limits_0^{\tau} N(\tau)\, d\tau$. In particular, if the force on the piston is constant $N(\tau) \equiv N_0$, the expression for the distribution of material porosity $\Pi(q, \tau)$ assumes the following form:

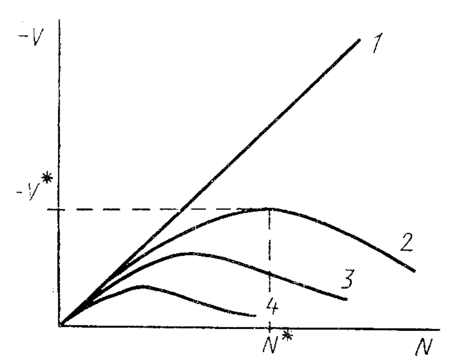

Fig. 1

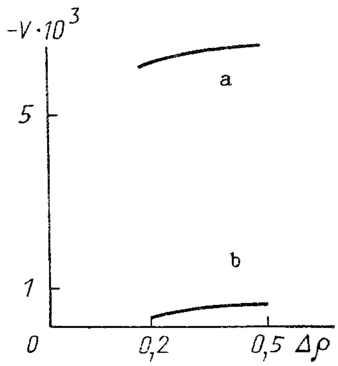

Fig. 2

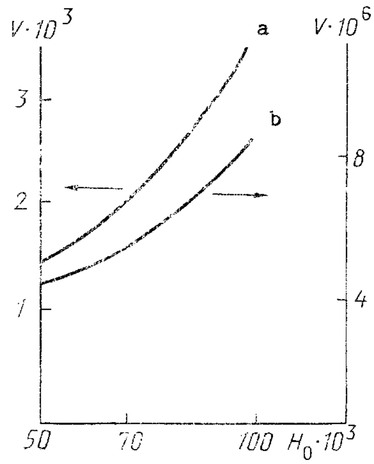

Fig. 3

Fig. 1. Velocity of piston as a function of applied force for various instants of time: 1) $\tau_1 = 0$; 2-4) $\tau_2 < \tau_3 < \tau_4$.

Fig. 2. Effect of the density difference $\Delta\rho$ on the velocity of the piston at various instants of time: a) $\tau = 0.5$; b) $\tau = 1$ sec. The force on the piston $N_0 = \text{const} = 10^8$ Pa, and the viscosity of the incompressible base $\eta_1 = 10^7$ Pa·sec. V, m/sec.

Fig. 3. Effect of the initial height of compression on the speed of the piston at a force $N_0 = 8 \cdot 10^7$ Pa, $\rho_u = 0.7$, $\rho_l = 0.5$, the density difference $\Delta\rho = 0.2$ for the time $\tau = 1$ sec: a) for $\eta_1 = 10^7$; b) for $\eta_1 = 10^{10}$ Pa·sec.

$$\Pi(q, \tau) = \Pi_0(q)\exp(-\tau/\tau_*), \tag{19}$$

where $\tau_* = 4\eta_1/3N_0$ is the characteristic compression time. We see that with the passage of time when $\tau > \tau_*$, regardless of the form of $\Pi_0(q)$, we encounter the effect of self-equalization in density, such as was discussed in [4]. Let us also note that according to (19) the characteristic time τ_* is identical for all individual volumes of the compressed material. It is convenient to use this relationship to determine the time of material formation prior to the specified residual porosity Π. Assuming that $\Pi \ll 1$ and holding the force to be constant, for the speed of the piston we obtain the following relationship:

$$V = -\frac{1}{\tau_*}\int_0^M \Pi_0(q)\,dq\exp(-\tau/\tau_*). \tag{20}$$

As we can see, the function V(N), just as $V(\tau_*)$, for fixed instants of time is nonmonotonic in nature (Fig. 1), which is a result of the competitive influence of the load factors and the relationship between the volumetric viscosity and the density. It is not difficult to understand that the appearance of the cofactors $\exp(-\tau/\tau_*)$ in relationship (20) comes about precisely when we take this last factor into consideration. In the function V(N) we can isolated two segments: in the first, the change in density and, consequently, the viscosity are insignificant and fundamental influences exerted by the increased load which causes a rise in velocity. In the second segment the influence of the increased density is more significant, since it increases the growth of the volumetric viscosity and, as a consequence, a rise in resistance to deformation. This leads to a reduction in velocity. With constant shearing and volumetric viscosities, this effect is absent and the velocity increases in direct proportion to the force. The nature of the correction factor introduced into the quantitative relationship V(N) owing to the initial distribution of density in the porous billet is determined by the physical sense of the integral $\int_0^M \Pi_0(q)dq$, which is nothing other than the mass of the material contained within the volume of the billet pores and exhibits a given initial nonuniformity.

The relationship of the initial distribution of density through the mass of the powder material introduces only a quantitative correction into the calculation of the function V(N). In particular, with a constant initial material density from (20) we obtain the familiar relationship

$$V(\tau) = -\frac{1}{\tau_*}(1-\rho_0)\rho_0 H_0 \exp(-\tau/\tau_*), \tag{21}$$

from which it is clear that the above-cited quantitative relationships governing compression pertain in this case as well. From the condition dV/dN = 0 we find the coordinates of the extremum points (V^*, N^*) in V(N):

$$N^* = 1/\tau_*, \quad V^* = -\frac{1}{e\tau}\int_0^M (1-\rho)\,d\rho.$$

Figure 1 shows the family of curves V(N) for various fixed instants of time τ, which in the given case should be treated as a parameter. The existence of a maximum value for velocity, dependent only on time and the initial density distribution, is a fundamental feature of the compression of compressed materials, governed by the relationships of the shearing viscosity and density. The described situation is entirely analogous to the one which prevails in nonisothermal flow of a fluid, when the strong nonlinear relationship between viscosity and temperature is brought about by the nonmonotonic nature of the dependence of resistance to deformation and the strain rate [9].

Specified Velocity Regime. In this case $V(\tau)$ is a known function and the force $N(\tau)$ has to be determined. For purposes of analyzing the function N(V) we will express $N(\tau)$ from relationship (16):

$$N(\tau) = -\frac{4}{3}\eta_1 V(\tau) F^{-1}(M). \tag{22}$$

Assuming that m = 1 and that the porosity $\Pi \ll 1$ from (19), we find

$$-\frac{3}{4\eta_1} N(\tau)\exp[-A(\tau)] = V(\tau)\int_0^M \Pi_0(q)\,dq.$$

This relationship can be rewritten in the form

$$\frac{d}{d\tau}\exp[-A(\tau)] = kV\tau, \quad k = \int_0^M \Pi_0(q)\,dq. \tag{23}$$

Integrating (23), we find that

$$\exp[-A(\tau)] = 1 + kV\tau. \tag{24}$$

Having substituted (23) into (22), we determined N(V):

$$N[V(\tau)] = -\frac{4\eta_1}{3}\frac{kV(\tau)}{1+kV(\tau)\tau}. \tag{25}$$

Since V < 0, k > 0, it follows from (24) that N(V) is a monotonic function, i.e., the increase in velocity for fixed instants of time lead to an increase in the force. This represents the fundamental difference in a regime with a given velocity from a regime with a given force in which the function V(N) is nonmonotonic.

From the standpoint of practical utilization, of interest is the theoretical formula for the determination of N(V) for a uniform distribution of the initial density. In this case, using (22) and carrying out transformations analogous to those presented above, we derive the relationship

$$N(V) = -\frac{4\eta_1}{3} V(\tau) M/(H_0 + V(\tau)\tau)(H_0 + V(\tau)\tau - M), \quad (26)$$

which clearly demonstrates the monotonicity of the function N(V).

Some Computational Results. Let us examine the effect of such parameters as the mass and dimensions of the compression operation, as well as the initial distribution of density here, insofar as these pertain to the relationship between the dynamic kinetic characteristics of the compression process. This relationship is expressed by the velocity of the piston as a function of the applied force V(N). The initial density distribution with respect to the coordinate q will be assumed in linear form $\rho_0(q) = a + bq$. When we take into consideration the density values ρ_u at the upper end of the billet and ρ_ℓ at the lower end of the billet we obtain the following form for the initial density distribution in the material being compressed:

$$\rho_0(q) = \rho_\ell\left(1 + \frac{\Delta\rho}{\rho_\ell}\bar{q}\right), \quad (27)$$

where $\bar{q} = q/M$; $\Delta\rho = \rho_u - \rho_\ell$ represents the density difference for the material. Let us note that a realistic density distribution may have a form different from (27). In such an event the assumed relationship may be treated as an interpolation of the nonlinear relationship. The proposed relationship $\rho_0(q)$ corresponds to the initial density distribution in Cartesian coordinates of the form $\rho_0(z) = \rho_\ell \exp[(\Delta\rho/M)z]$. The assumed form of $\rho_0(q)$ allows us explicitly to find F(M) from (16) and this makes it possible to trace the influence exerted by the initial nonuniformity of the material. This influence on the operational parameters of the process has been investigated on compressed materials of identical mass and volume, i.e., for specimens with a constant average density. Figure 2 shows the result from a calculation of piston speed which corresponds, according to (18), to the force $N_0 = 10^8$ Pa as a function of the magnitude of the original density difference. We see that for various instants in the compression process (curves a and b correspond to 0.5 and 1 sec with a characteristic compression time $\tau_* = 2$ sec) a change in the density difference from 0.2 to 0.5 has virtually no effect on the speed of the piston. Unlike the density difference, the mass and dimensions of the materials being compressed significantly affect the operational characteristics of the process. Figure 3 shows the speed of the piston as a function of the initial height of the compressed materials, and this varied from 50 to 100 mm at a constant billet mass with a specific density difference $\Delta\rho/H_0$. Calculations showed that for selected parameters of the process the change in the initial dimensions of the compressed materials within the indicated limits leads to a significant increase in velocity: by a factor of 2-3. An analogous result is obtained in calculating piston speed as a function of the varying compression mass whose volume and form of density difference are identical at the instant of compression: the increase in billet mass from 0.03 to 0.1 kg results in a linear increase in piston speed by a factor of 2-3.

NOTATION

t, z, time and instantaneous height (Euler coordinates); τ, q, time and instantaneous mass (Lagrange coordinates); ρ, density of the material relative to the density of the uncompressed base; M, billet mass; H_0, initial height of the compressed material; σ_r, σ_θ, σ_z, stress components; v, velocity component in longitudinal direction; η and ξ, shearing and volumetric viscosity; η_1, viscosity of uncompressed base; $N(\tau)$, piston force; $V(\tau)$, piston speed; Π_0 and Π, initial and instantaneous material porosity; ρ_0, initial density of compressed material; τ_*, characteristic compression time; ρ_u, ρ_ℓ, density at upper and lower ends of compressed material; ρ_1, density of uncompressed base; m, exponent in empirical relationships for viscosity.

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